The gyroscope fully understood: Complete gyroscopic motion with external torque

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Abstract

The series of papers on gyroscopes is completed by the full Lagrangian theory of gyroscopes including external torque. Numerical solutions are presented. An approximation for fast rotating gyroscopes is introduced which reduces the calculational effort significantly. The lifting effects observed by Laithwaite and Kidd are explained quantitatively. These, as well as Shipov's experiments, can be interpreted on basis of classical mechanics. The solutions of Lagrange equations prove the existence of a local linear momentum of driven gyroscopes that can be used for propulsion. An own experimental evaluation of lifting effects is under way.

Keywords: gyroscope; Lagrangian mechanics; Laithwaite experiment; Kidd experiment; Shipov experiment.

1 Introduction

The laws of gyroscopic motion have been a mystery to many people for centuries because rotational systems are not easy to understand. A gyro reacts to applied forces by a motion perpendicular to that forces. In classical mechanics only the Lagrange formalism is able to completely describe this type of rotational motion. The laws of gyroscopic motion are difficult to set up because it is a theory of rigid bodies [5]. Actually there are two problems. First, the equations of motion are complicated so that it is cumbersome to derive them by hand. Therefore in textbooks nowhere the full equation set (with all coordinates) is given for a complete motion in 3D. Second, analytical solutions do only exist even for subsets of the equations. Therefore, for the general equation set which is presented in this paper, no analytical solutions exist. The equations have to be solved numerically on a computer. Such solutions are rarely found in textbooks, in particular not for problems of classical mechanics. This is a subject of theoretical physics which obviously is dominated by very conservative physicists.

Besides the field of serious physical research, there is a plain field of more or less unscientific assertions about gyroscopes effused by hobby researchers and

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inventors. This may have led to the opinion in the scientific world that there is "nothing new to get" about gyroscopes. However, in this article we show that some aspects of gyroscopes have not been fully researched and understood. With aid of computer algebra and numerical solution methods we were able to present the full equation set without approximations and present numerical solutions. By Lagrangian theory we have the means for computing the complete dynamics of classical systems. This has obviously never been done for gyroscopes to such detail, in particular when external torque is applied. As a result, we find a new method of generating a linear momentum locally in a system. This corresponds to a black box that is able to effect its own propulsion without interaction with its environment.

In section 2 we give a summary on our earlier papers on this subject and add an approximation for fast rotating gyros. Using external torque leads to lifting effects, which confirm the findings of Laithwaite and Shipov. After a short outlook on our own experiments (section 3) we discuss the results in section 4.

2 Theory of the gyroscope

2.1 The gyroscope with one point fixed

We compute the motion of a symmetric top with one point fixed, for example a spinning top on a table. This is as already worked out in [1–3], a Lagrangian formulation based on ECE2 theory [4]. The fixed point is assumed the centre of a coordinate system consisting of three Eulerian angles, see Fig. 1. The angles θ and ϕ are identical to those of a spherical coordinate system (polar and azimuthal angle). ψ is the rotation angle around the body axis of the gyro. The kinetic energy (defined by the body coordinates) is purely rotational. According to the Lagrange calculus, the body coordinates are to be transformed to the (θ, ϕ, ψ) coordinate system, leading to the rotational kinetic energy

$$T_{rot} = \frac{1}{2} I_{12} \left(\dot{\phi}^2 \sin(\theta)^2 + \dot{\theta}^2 \right) + \frac{1}{2} I_3 \left(\dot{\phi} \cos(\theta) + \dot{\psi} \right)^2 \tag{1}$$

where I_{12} and I_3 are the moments of inertia around the three principle gyro axes (for details see [1–3, 5]). The potential energy is defined from the gravitational field at the earth's surface:

$$U = m\, g\, Z = m\, g\, h \cos(\theta) \tag{2}$$

with constant gravitational acceleration g. The Lagrangian is

$$\begin{aligned} \mathscr{L} &= T_{rot} - U \\ &= \frac{1}{2} I_{12} \left(\dot{\phi}^2 \sin(\theta)^2 + \dot{\theta}^2 \right) + \frac{1}{2} I_3 \left(\dot{\phi} \cos(\theta) + \dot{\psi} \right)^2 - m\, g\, h \cos(\theta). \end{aligned} \tag{3}$$

The three Euler-Lagrange equations for the angular coordinates q_j

$$\frac{\partial \mathscr{L}}{\partial q_j} = \frac{d}{dt} \left(\frac{\partial \mathscr{L}}{\partial \dot{q}_j} \right) \tag{4}$$

lead to three equations containing first and second time derivatives of the angular coordinates and can be re-arranged giving the ordinary differential equation system

$$\ddot{\theta} = \frac{\left((I_{12} - I_3)\, \dot{\phi}^2\, \cos(\theta) - I_3\, \dot{\phi}\, \dot{\psi} + mgh\right)\, \sin(\theta)}{I_{12}}, \tag{5}$$

$$\ddot{\phi} = -\frac{\left((2I_{12} - I_3)\, \dot{\phi}\, \cos(\theta) - I_3\, \dot{\psi}\right)\, \dot{\theta}}{I_{12}\, \sin(\theta)}, \tag{6}$$

$$\ddot{\psi} = \frac{\left((I_{12} - I_3)\, \dot{\phi} \cos(\theta)^2 + I_{12}\dot{\phi} - I_3\, \dot{\psi}\, \cos(\theta)\right)\, \dot{\theta}}{I_{12}\, \sin(\theta)}. \tag{7}$$

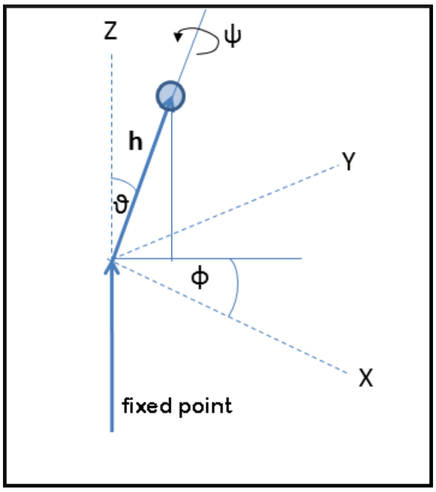

Figure 1: Eulerian angles of a gyro with one point fixed.

These equations can be solved numerically in principle. There is however some more information contained in the Lagrange equations (4). There are two constants of motion [1] representing the angular momenta around the Z axis and body axis:

$$L_\phi = I_{12}\, \dot{\phi} \sin(\theta)^2 + I_3\, \cos(\theta)\, \left(\dot{\phi}\, \cos(\theta) + \dot{\psi}\right), \tag{8}$$

$$L_\psi = I_3\, \left(\dot{\phi}\, \cos(\theta) + \dot{\psi}\right). \tag{9}$$

These equations contain only the first time derivatives of ϕ and ψ. Using these equations instead of Eqs. (6, 7) leads to the simpler differential equation system

$$\ddot{\theta} = \frac{\left((I_{12} - I_3)\, \dot{\phi}^2\, \cos(\theta) - I_3\, \dot{\phi}\, \dot{\psi} + mgh\right)\, \sin(\theta)}{I_{12}}, \tag{10}$$

$$\dot{\phi} = \frac{L_\phi - L_\psi\, \cos(\theta)}{I_{12} \sin(\theta)^2}, \tag{11}$$

$$\dot{\psi} = \frac{L_\psi - I_3\, \dot{\phi}\, \cos(\theta)}{I_3}. \tag{12}$$

The constants L_ϕ and L_ψ have to be chosen appropriately for a solution. Also for this simpler equation system a numerical solution mechanism is required.

I_{12}	$h^2\,m = 0.013$ kg m^2
I_3	0.005552 kg m^2
m	1.3 kg
g	9.81 m/s^2
h	0.08 m
L_ϕ	0.015 kg m^2/s
L_ψ	0.3 kg m^2/s
$\dot{\psi}_0$	54.03 rad/s
f	516.0 /min
T_{q0}	0.2 Nm
D	1.2 kg m^2/s^2
θ_0	$\pi/2$

Table 1: Gyro parameters.

2.2 Approximation for fast rotation

In case of high rotation speed $\dot{\psi}$ the angular momentum L_ψ is much higher than L_ϕ and, consequently, $\dot{\psi}$ is much greater than $\dot{\phi}$. This leads to motion on different time scales, we obtain a so-called stiff system. Such a system is difficult to handle numerically because small time steps have to be used to model the fast rotation while nearly no change is visible in the slow rotation variables. Therefore we approximate Eq. (12) by

$$\dot{\psi} \approx \frac{L_\psi}{I_3} = \text{const.} \tag{13}$$

Inserting this constant value in the kinetic energy (1) then leads to only two (slowly varying) Lagrange variables θ and ϕ. There is only one constant of motion left for ϕ. The Lagrangian (3) leads to the final equation set:

$$\ddot{\theta} = \frac{\Big((I_{12} - I_3)\ \dot{\phi}^2\ \cos(\theta) - I_3\,\dot{\phi}\,\dot{\psi}_0 + mgh\Big)\ \sin(\theta)}{I_{12}}, \tag{14}$$

$$\dot{\phi} = \frac{L_\phi - I_3\dot{\psi}_0\ \cos(\theta)}{I_{12}\sin(\theta)^2 + I_3\cos(\theta)^2} \tag{15}$$

with a constant $\dot{\psi}_0$ to be defined a priori. If the rotation frequency f is given in rpm (rounds per minute), it is

$$\dot{\psi}_0 = 2\pi f/60 \tag{16}$$

in radians/s. Numerical tests showed that for $f \gtrsim 500$/min there is no visible difference between this approximation and the exact method.

Examples of a fast gyro are given in Figs. 2-3. The calculations were carried out with the parameters listed in Table 1. From Fig. 2 can be seen that ψ increases linearly and much faster than ϕ, we have a fast rotating top with $\dot{\psi} \approx$ const. ϕ and θ are nearly linear with small oscillations. The oscillations of θ correspond to a periodic nutation as can be seen from the space curve of the centre of mass graphed in Fig. 3.

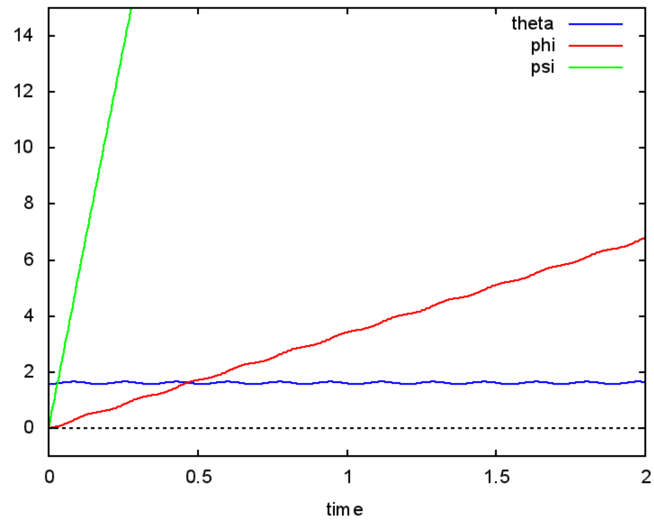

Figure 2: Time dependence of θ, ϕ, ψ for free running gyro. This is identical to a fast running gyro with predefined $\dot{\psi}$.

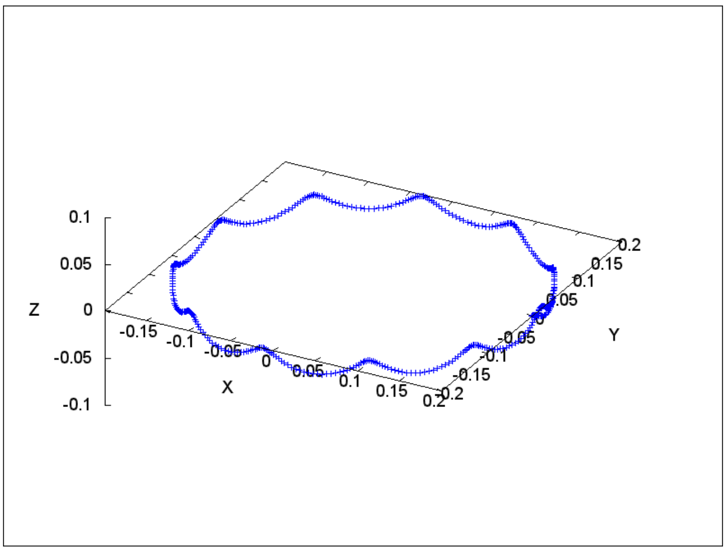

Figure 3: Space curve of centre of mass for a free (and fast) running gyro.

2.3 Adding external torque

An external torque can be introduced by a generalized force into the Lagrange mechanism. Since Lagrange theory works with potentials, we define a potential giving a constant torque T_{q0} around the Z axis (for the angle ϕ) by

$$T_q = -\frac{\partial U_q}{\partial \phi} \tag{17}$$

with

$$U_q = -T_{q0}\,\phi \tag{18}$$

and add this to the potential energy:

$$U = mgZ + U_q = mgh\cos(\theta) - T_{q0}\,\phi. \tag{19}$$

Then there is no constant of motion left and the Euler-Lagrange equations read (in fast rotation approximation):

$$\ddot{\theta} = \frac{\left((I_{12} - I_3)\ \dot{\phi}^2\ \cos(\theta) - I_3\,\dot{\phi}\,\dot{\psi}_0 + mgh\right)\ \sin(\theta)}{I_{12}}, \tag{20}$$

$$\ddot{\phi} = -\frac{\left(2\,(I_{12} - I_3)\ \dot{\phi}\ \cos(\theta) - I_3\,\dot{\psi}_0\right)\ \sin(\theta)\,\dot{\theta} - T_{q0}}{I_{12}\sin(\theta)^2 + I_3\cos(\theta)^2}. \tag{21}$$

Because there is no constant of motion, both equations are of second order. The torque term – if chosen not too small – has an enormous impact on the motion of the gyroscope. As had been shown in [2] for a slowly rotating gyroscope, the results can be very exotic in dependence of the value of T_{q0} and the initial conditions.

Other, more complicated effects emerge when T_q is made periodic in time, for example

$$T_q = T_{q0}\cos(\omega t) \tag{22}$$

with a time frequency ω. Then new effects like heterodynes in angular velocities can appear [2]. In this case there is no continuous rotation in ϕ direction. By suitable initial conditions, it is even possible to stop all rotations.

Some examples, based on parameters listed in Table 1, are graphed in Figs. 4-6. A constant driving term T_q leads to a linear rise of the spinning top until it reaches its "pole position" ($\theta = 0$ (Fig. 4). The top is able to counterbalance the driving torque in ϕ direction by a perpendicular motion corresponding to the gyro laws. After having reached the pole position, acceleration is directly applied to the ϕ angle, showing a quadratic increase of the angle.

These effects are also detectable by considering the angular velocities graphed in Fig. 5. The ϕ velocity increases linearly after the top is no more able to lift. The θ velocity oscillates with average value below zero, i.e. there is a net motion of θ to smaller angles. After reaching the pole position $\theta = 0$, there is a pure oscillation. This can also be seen from the centre of mass curve in Fig. 6. The rising of the top is superimposed by nutation-like oscillations.

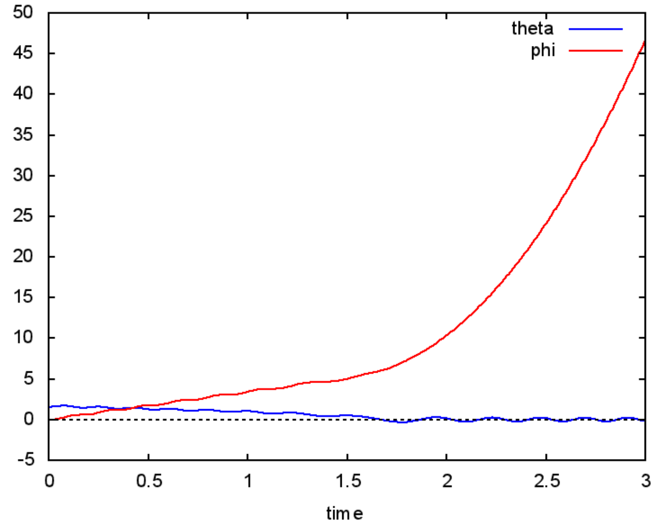

Figure 4: Time dependence of θ, ϕ for a fast running, driven gyro.

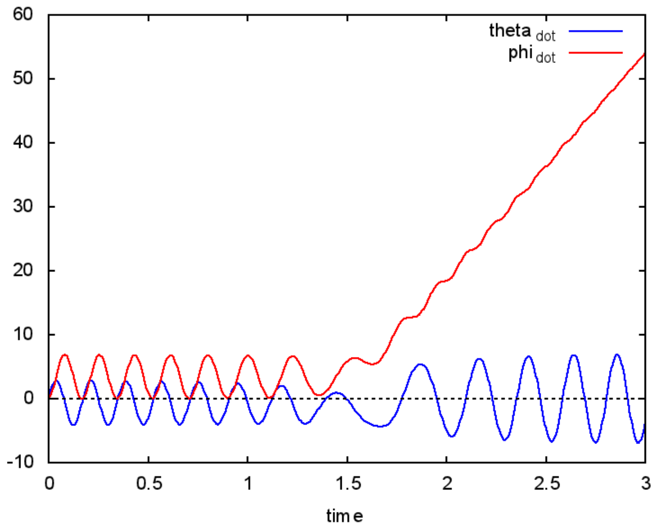

Figure 5: Time dependence of angular velocities $\dot{\theta}$ and $\dot{\phi}$ for a fast running, driven gyro.

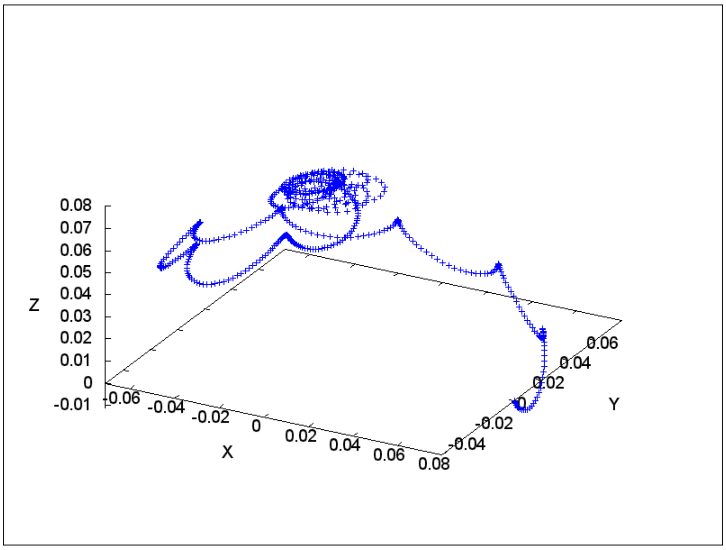

Figure 6: Space curve of centre of mass for a fast running, driven gyro.

2.4 Lifting effects

2.4.1 Blocking lifting by a spring

A lift of the gyroscope with the axis fixed at one point point means that the spinning top aligns in Z direction (perpendicular to the ground). Then the angle θ goes to zero as demonstrated in Fig. 6. If this lifting is confined by a mechanical mechanism, it should be possible to transfer the lifting force to the environmental mechanics of the gyro. Since we cannot simply impose nonlinear bounds to the range of θ in Lagrange theory (steady functions are required), we introduce a spring mechanism in a preliminary approach that damps the uplift of the gyro. For this, we introduce an unharmonic term

$$U_1 = D(\theta_0 - \theta)^2 \tag{23}$$

in the potential energy with a spring constant D and $\theta_0 = \pi/2$. Then the Euler-Lagrange equation (20) is modified by an additional term of an external retarding torque:

$$\ddot{\theta} \to \ddot{\theta} + 2\frac{D}{I_{12}}(\theta_0 - \theta). \tag{24}$$

The example of the preceding section then shows a slower lifting of the rotating mass and the highest point with $\theta = 0$ is not reached, see Fig. 7.

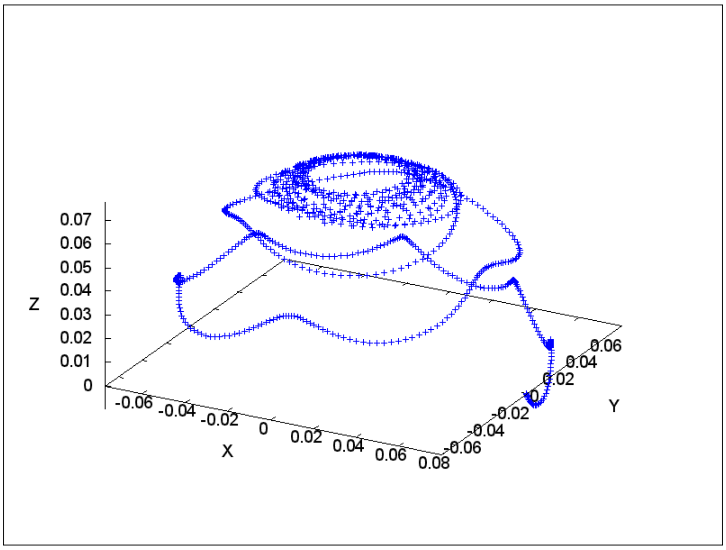

Figure 7: Space curve of centre of mass for a fast running, driven gyro with spring break in θ direction.

2.4.2 Free-falling gyro

In order to study true lifting effects we make the gyro freely moving in the Z direction. We use the fast gyro approximation with $\dot{\psi} = \dot{\psi}_0$ =const as before, and the gyro is driven by a constant torque T_{q0} around the Z axis. For vertical motion we have to introduce the Lagrange coordiante Z, see Fig. 8. We do not allow any motion in the polar θ direction, leaving this angle at 90 degrees by definition. This configuration is used for example in the patent of Kidd [9, 10] and reduces the number of Lagrange variables, leaving only ϕ and Z.

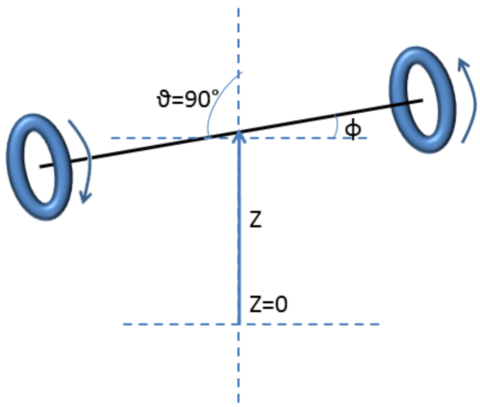

Figure 8: Free falling, driven gyro.

Besides the rotational kinetic energy of Eq. (1), we now have to add a

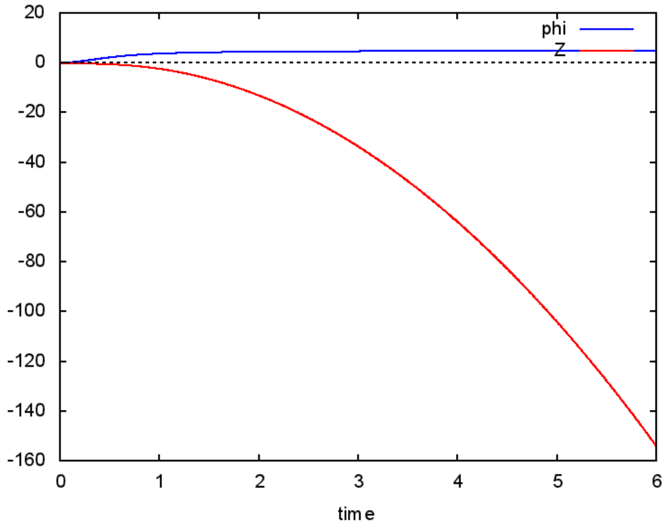

Figure 9: Angle and height development of a free falling, driven gyro with $\dot{\psi}_0 = 516/\text{min}$.

translational kinetic energy term

$$T_{\text{trans}} = \frac{m}{2} \dot{Z}^2 \tag{25}$$

to the Lagrangian (3). The resulting Euler-Lagrange equations are:

$$\ddot{\phi} = -\frac{2 I_3 Z \, \dot{Z} \, \dot{\phi} - T_{q0} h^2 + I_3 \, \dot{Z} \, \dot{\psi}_0 \, h}{I_{12} \, h^2 + I_3 \, Z^2}, \tag{26}$$

$$\ddot{Z} = \frac{I_3 Z \, \dot{\phi}^2 + I_3 \, \dot{\psi}_0 h \, \dot{\phi} - g \, h^2 m + 2 D_Z \, (Z_0 - Z) \, h^2}{h^2 m}. \tag{27}$$

For the simulation we have assumed two synchronously running gyros with total mass m, i.e. each gyro now has the mass $m/2$.

The results with parameters used as in Table 1 show a gyro in free fall, obviously no lifting. As can be seen from Fig. 9, the Z coordinate of the gyro goes down parabolically as in free fall. This is what one would naively expect. The ϕ angle comes to rest although there is a driving torque on it. In free fall the gyroscope is force-free and there is no precession [2]. Obviously the applied torque is not strong enough to maintain precessional motion. The type of motion changes completely, however, when the rotation speed of the wheels is increased, for example from 516/min to 1032/min (Fig. 10). By the graphs in Fig. 10 we indeed observe a lifting of the gyroscope, overlaid with a small oscillatory motion. The angle ϕ increases linearly, indicating a constant angular velocity. Inspection of the simulational results shows that the oscillations depend intricately on initial conditions. Also the point where fall changes to lift depends on these; the system here shows a sensitive behaviour.

Lagrange theory is based on energy conservation by definition. Therefore energy is conserved in all closed systems modeled by Lagrange theory. If energy is added by external sources (a torque in our case), this should appear as an

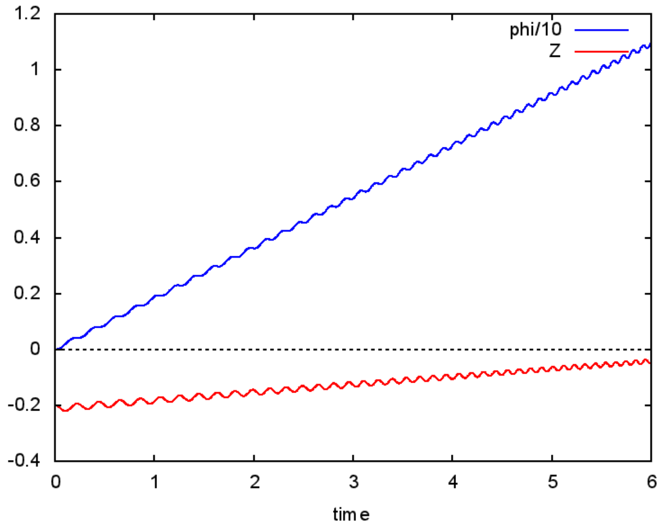

Figure 10: Angle and height development of a free falling, driven gyro with $\dot{\psi}_0 = 1032/\text{min}$.

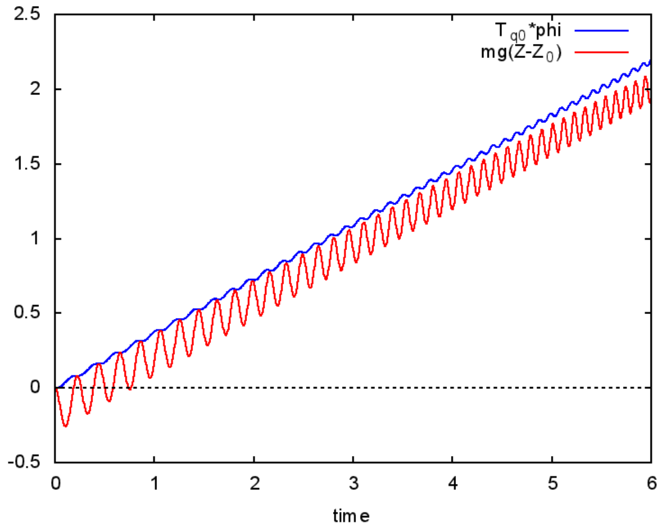

Figure 11: Potential energies of a lifting, driven gyro with $\dot{\psi}_0 = 1032/\text{min}$.

extra energy. In the case of the self-lifting gyro this extra energy should appear in the potential energy mgZ raising the gyro. The torque is modeled as a rotational potential energy $T_{q0}\,\phi$. In case of perfect energy conservation, we should have

$$m\,g\,(Z - Z_0) = T_{q0}\,\phi. \tag{28}$$

We have added a constant shift Z_0 so that both potential energies start at zero for $t = 0$. Both expressions are graphed in Fig. 11. Obviously the difference is near to zero but not exactly zero and oscillating. However this is not because Lagrange theory fails but is an artifact of modeling. Remember that we used the constant angular velocity approach (Eqs. 13/16)

$$\dot{\psi}_0 = \text{const} \tag{29}$$

for the gyro wheels. From earlier papers we know that the motions of all gyro coordinates are coupled. So we assumed that the gyro speed is held at a constant value by an additional driving force, and so did we in the experiments (see later). If we would do a 3-coordinate calculation, the difference would be exactly zero. In conclusion, the energy for linear transport of the gyro is taken from the two input torques.

2.5 Re-evaluating the experiments of Laithwaite and Shipov

In [2] we already made an attempt to explain the experiments of Laithwaite [6] and Shipov [7] qualitatively with a slowly rotating gyro. By applying a torque in ϕ (i.e. around the Z direction) a gyroscope should lose weight. We showed by our numerical calculations that this is indeed possible. A sufficiently fast rotating gyro is required. Laithwaite rotates a more than 20 kg gyro by one hand and thus gives it an angular acceleration in precessional direction. He was ridiculed by his colleagues. Alexander Kidd constructed a twin gyro [9,10] from that we derived our design. His construction could not be explained by the University of Southampton. This paper proves on a theoretical level that both handlings operate as claimed, although it seems not plausible at a first glance.

Shipov has done research on gyroscopes over years and developed a complete theory named torsion physics [8]. He investigated linear propulsive motion. His constructions are different from those of Kidd. Shipov and his colleagues found irregular changes in momentum which probably can be explained by the additional linear momentum reported in this paper.

3 Own experiments

The Munich group started its own experiments with a twin gyro (photo in Fig. 12) similar to Kidd's construction sketched in Fig. 8. One motor is used for driving the wheels, a second motor generates the ϕ torque. After some experimenting, the belt transmission was replaced by gear wheels to avoid too high mechanical losses. The rotating arm was put on a spring so that a lifting can be observed. In first experiments we found a lifting qualitatively. Further experiments with measurements and improvements of the apparatus are scheduled.

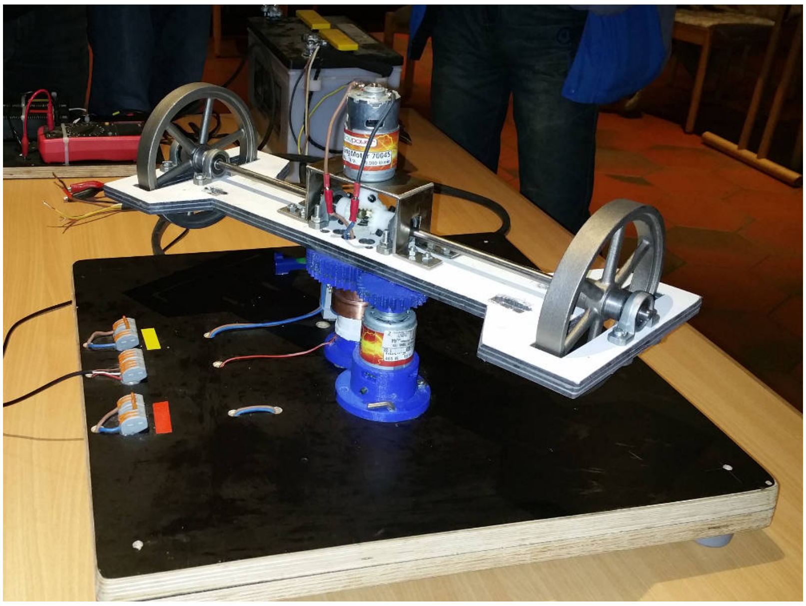

Figure 12: Experimental setup of the double gyro.

4 Summary and discussion

A comprehensive analysis of gyroscopic motion was given in this paper. A new approach for fast moving gyroscopes was developed. The most astonishing aspect is the possibility of lifting. The local torques generate a linear momentum, therefore this is not conserved. This momentum is not a constant of motion, while the corresponding coordinate is part of the Lagrangian formulation. So there is no need for expecting such a conservation. The reader should bear in mind that Lagrange theory uses *generalized coordinates* and *generalized momenta*, which are linear and angular momenta. Which ones are relevant, depends on the coordinates.

The problem in understanding the behaviour of gyroscopes is from history. Newton was not familiar with the difference between linear and angular momentum. He formulated his laws for linear momentum and each physicist takes this for the truth down to the present day. However, Euler and Lagrange introduced the dynamics of rotational motion into physics, and Newton's third law (conservation of momentum) should be reformulated in the way that only the momenta characteristic for a specific motion are conserved. This is true for Lagrangian theory, but not for Newton's original exclusive linear momentum. So energy conservation does not mean that any linear momentum must be conserved. The exhaustive scientific building of mechanics is based on Euler, Lagrange and Hamilton, not on Newton.

Although Shipov did a lot of theoretical and practical research on gyroscopic motion, he missed the simplest explanation given here by classical Lagrange theory. Numerical solutions of equations of motion are available since the early 1980ies so it is astonishing that he felt compelled to develop a very complex torsion theory for the motion of gyroscopes [8]. In this paper we can explain lifting as a pure classical effect.

The linear momentum generation by gyroscopes can have practical application in positioning of satellites. Currently gyros are used to stabilize positions, but change of orbits without need for fuel would be possible. For applications on the earth surface, the heavy weight of gyroscopes is a massive drawback. Nevertheless a hovering above ground may be possible. Simulation result show that the transition point from free fall to lifting motion is difficult to meet. For hovering it would be required to stabilize the angular velocities of ϕ and/or ψ in a way that Z remains constant. With the parameters listed in Table 1, we had to increase $\dot{\psi}_0$ by a factor of exactly 1.1255, obviously the system is very sensitive in this range and such a state can only be maintained by a digital control process.

Acknowledgment

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